Complex Analysis, MTH G204.
Fall 2003. Professor Mikhail Shubin.

Textbook:

Complex Analysis, by Theodore W. Gamelin. Springer-Verlag New York, Inc., 2001.

Office: 460 Lake Hall. **Phone:** (617)373-5676 **E-mail:** shubin@neu.edu

Class meetings: Tuesday and Thursday 5:50 – 7:20 p.m. 544 Nightingale Hall

Complex analysis is fascinatingly beautiful and, at the same time, crucially important for mathematics and its applications. No other domain of mathematics can boast such a jewel as the Euler formula $e^{i\pi} = -1$. (Consider that e comes from compounded interest, π is the area of the disc with a radius of 1, and $i = \sqrt{-1}$.)

Applications of Complex Analysis are abundant. They include calculation of improper integrals (which can not be calculated by the real analysis tools), solving some problems in fluid dynamics, establishing the famous asymptotic law of distribution of primes (i.e. finding an approximate formula for the nth prime number), introducing new tools in general relativity, and many others.

The course will cover the following topics:

1. Elementary functions and conformal maps defined by them.
2. Definition and simplest properties of analytic functions. Harmonic functions. Their relationship to analytic functions. The Cauchy–Riemann equations.
3. Line integration and complex integration. Cauchy's integral formula and its simplest applications. Liouville's theorem.
4. Power series and their properties. Power series expansion of analytic functions. The Laurent decomposition of analytic functions. Isolated singularities.
5. The residue calculus and calculation of integrals.
6. The argument principle and Rouché's theorem. Counting zeros of polynomials and analytic functions. Open mapping and inverse function theorem.
7. The Schwarz reflection principle and its applications.
8. Conformal mappings. The Riemann mapping theorem.

Some more advanced topics, also covered in the book, may be offered as projects. (Examples: Julia sets and the Mandelbrot set, infinite products, the Gamma function, the Zeta function and the Prime Number Theorem.)

The prerequisites include an analysis course with elements of topology. For example, Chapters 1-9 from "Principles of Mathematical Analysis" by W.Rudin are sufficient (but not all the material there is necessary).

Gamelin's textbook is very well written and contains a lot of exercises with answers, hints and, in some cases, solutions. Some homework will be assigned weekly, collected, and graded. It is recommended that you do as many of the exercises from the book as possible (whether assigned as homework or not). You shall profit greatly from this.

The grade will be based on homework assignments, the project, and, possibly, the final exam.