MIKHAIL A. SHUBIN
CURRICULUM VITAE

Updated January 2006

1 EDUCATION AND DEGREES

Doctor of Science in Physics and Mathematics, Leningrad branch of Steklov Mathematical Institute of Academy of Sciences of USSR (LOMI), 1981.

Ph.D. in Mathematics (Differential Equations), Department of Mech. and Math. Moscow State University, 1969.

Master degree in Mathematics, Department of Mech. and Math. Moscow State University, 1966.

2 EMPLOYMENT HISTORY

Department of Mathematics, Northeastern University, 1992 -

Department of Mathematics M.I.T., 1991 - 1992

Institute of New Technologies (Moscow), 1990 - 1991

Department of Mechanics and and Mathematics, Moscow State University, 1969 - 1990

3 HONORS

Member of Russian Academy of Natural Sciences (elected in 1996).

Matthews Distinguished University Professor, Northeastern University (from 2001).

4 GRANTS

1. DMS-0107796, 04/01/2001 - 03/31/2005, National Science Foundation
2. DMS-9706038, 07/01/1997 - 06/30/2000, National Science Foundation.
3. BSF-94-00299, 09/01/95 - 09/01/98, USA - Israel Binational Science Foundation, joint with Michael Farber (Tel Aviv University, Israel) and Jerome Levine (Brandeis University).
4. DMS-9222491, 01/01/93 - 07/31/96, National Science Foundation.

5 PhD THESES DIRECTED

Recipient	Subject	Date
S.Smagin	Complex powers of hypoelliptic operators in $\mathbf{R}^n$	1974
A.Gusev	Density of states for elliptic operators	1977
V.Kiseljov	Almost periodic Fourier integral operators	1978
V.Bezjaev	Some spectral properties of hypoelliptic operators	1979
T.Bogorodskaja	Density of states for random pseudo-differential operators	1983
G.Meladze	Pseudo-differential operators on unimodular Lie groups	1984
M.Efendiev	Partial differential equations in Besikovich spaces	1985
Yu.Kordyukov	Elliptic operators on manifolds of bounded geometry	1987
L.Malozjomov	Spectral theory of Schrödinger operators with random and asymptotically almost periodic coefficients	1988

A.Volovoi	Estimate of remainder in two-term asymptotics of distribution function for eigenvalues of elliptic operators on closed manifolds	1988
D.Efremov	Spectral asymptotics of elliptic operators on hyperbolic space	1988
A.Efremov	Lefschetz-type theorems for elliptic complexes on non-compact manifolds	1989
I.Bondareva	Increasing solutions of KdV-type equations	1989
I.Oleinik	On the essential self-adjointness of the Schrödinger type operators on complete Riemannian manifolds	1997
O.Milatovic	On the essential self-adjointness of the Schrödinger type operators on Riemannian manifolds	2002
S.Dubrovsky	Differential invariants of geometric structures	2004
J.Perez	L^2-Fredholm property of $\bar{\partial}$ Neumann problem on strongly pseudoconvex G-manifolds	2005

6 LECTURE COURSES

A. In Moscow State University:

1. Partial differential equations (general course).
2. Hyperfunctions and their applications.
3. Pseudo-differential operators and spectral theory.
4. Fourier integral operators.
5. Introduction to microlocal analysis.
6. Distributions and their applications.
7. Introduction to spectral theory of operators.

8. Selected problems of modern analysis and mathematical physics.
9. Mathematical methods of quantum mechanics.
10. Spectral theory of Schrödinger operators.
11. Almost periodic functions and partial differential equations.
12. Introduction to non-standard analysis and its applications.
13. Introduction to elliptic topology.
14. Supersymmetry and index theory of elliptic operators.
15. Von Neumann algebras and non-commutative integration.

B. In other institutions.

1. Pseudo-differential operators and their applications (Institute of Mathematics of Tajic Academy of Sciences; Erevan State University).
2. Elliptic operators with almost periodic coefficients (Voronezh Institute of Forest Engineering; Voronezh All-Union Winter Mathematical School).
3. Nonstandard analysis and singular perturbations of ordinary differential equations (Donetsk State University; Lvov State University; Voronezh All-Union Mathematical School).
4. Index of elliptic operators (Voronezh All-Union Winter Mathematical School).
5. Elliptic operators and von Neumann algebras (Odessa State University).
6. Spectral theory of elliptic operators on non-compact manifolds (Summer School on Semi-classical Methods, Nantes).
7. Von Neumann algebras and their applications (M.I.T.).
8. Differential equations (Northeastern University).

9. Topology-1 (Northeastern University).
10. Partial differential equations - 1 (Northeastern University).
11. Geometry-2 (Northeastern University)
12. Differential geometry (Northeastern University)
13. Spectra near zero in topology (Winter School in Les Diablerets, Switzerland)
14. Functional analysis (Northeastern University)
15. Riemannian geometry and general relativity (Northeastern University)
16. Complex analysis in several variables (Northeastern University)
17. Spectral theory of the Schrödinger operators on non-compact manifolds (Instructional Conference "Spectral Theory and Geometry", University of Edinburgh, 1998)
18. L^2-holomorphic functions on non-compact manifolds (Cinvestav, Mexico)
19. Index of elliptic operators (Northeastern University)
20. Fundamentals of Analysis (Northeastern University)
21. Basics of Analysis (Northeastern University)
22. Von Neumann algebras and L^2 invariants in geometry and topology (Northeastern University)
23. Capacities and their applications (Humboldt University, Berlin)
24. Von Neumann algebras and L^2 invariants (Independent University of Moscow)
25. Can one see the fundamental frequency of a drum? (Instructional Conference in Analysis and Geometry, Novosibirsk, Russia, 2004)
26. Complex Analysis (Northeastern University)

27. Geometry-1 (Northeastern University)

28. Capacity in spectral theory of Laplace and Schrödinger operators.
    Frontier Lectures in Texas A&M University, 2005;
    Pathways Lectures, Keio University, Tokyo, Japan, 2005;
    Summer School "Analysis and Mathematical Physics", Cuernavaca, Mexico, 2005

7 INVITED TALKS AND LECTURES AFTER 1987

1. Conference on Elliptic Operators on Singular and Non-compact Manifolds, Oberwolfach, 1987.

2. Conference on Pseudo-differential Operators, Oberwolfach, 1989 and 1991.

3. University Paris-7, 1988 and 1989.

4. University Paris-Sud, 1989.

5. Ecole Normale Superieure, 1989.

6. Ecole Polytechnique, 1989, 1999

7. College de France, 1989.

8. University Paris-Nord, 1989.

9. University of Marseille, 1989.

10. University of Nantes, 1989.

11. Mathem. Institut der Technischen Hochschule, Darmstadt, 1989, 1992.

12. University of Bochum, 1989, 1994.

13. Banach Centrum, Warsaw (Poland), 1990.

14. International Workshop "Analysis in Domains and on Manifolds with Singularities", Breitenbrunn, 1990.

15. University of Augsburg, 1990, 1992, 1994.
16. Conference “25 years of Microlocal Analysis”, Irsee, 1990 (plenary talk).
17. Karl-Weierstrass-Institut fur Mathematik, 1990.
18. Freie Universität, Berlin, 1990.
19. Ohio State University, Columbus, Ohio, 1991, 1994.
20. University of Maryland, College Park, 1991 (Colloquium); 1992, 1993, 1994.
21. State University of New York at Buffalo, 1991 (Colloquium).
22. Indiana University – Purdue University, Indianapolis, 1991 (Colloquium), 1996 (2 lectures and Colloquium).
23. University of Georgia, Athens, 1991 (Colloquium).
24. Wichita State University, Wichita, KA, 1991 (Boeing lecture).
25. University of California at Los Angeles, 1991 (Colloquium), 1992, 1996.
26. University of California at Irvine, 1991 (Colloquium).
27. University of California at Berkeley, 1991 (Colloquium).
28. University of California at Santa Cruz, 1991 (Colloquium), 1995 (Colloquium).
29. University of Southern California, Los Angeles, 1991 (Colloquium).
30. Stanford University, 1991 (Colloquium), 1995.
31. Massachusetts Institute of Technology, 1991, 1992, 1993, 1994, 1995, 1996, 1997, 1998, 1999, 2000, 2001, 2002.
32. The John Hopkins University, Baltimore, 1991 (Colloquium).
33. Courant Institute of Mathematical Sciences, New York University, 1991, 1992, 1993, 1998, 1999, 2001, 2003

34. Princeton University, 1991.

35. Rutgers University, 1991 (Colloquium), 1994.

36. State University of New York at Stony Brook, 1991 (Colloquium).

37. Graduate Center of City University of New York, 1991, 2000.

38. Colloque International sur les Methodes Semiclassiques (Nantes), 1991.

39. ETH Zürich, 1991, 1992, 1993, 2003, 2005

40. Cornell University, 1991.

41. Temple University, 1991 (Colloquium).

42. University of Toronto, 1992 (Colloquium), 1998, 2000.

43. University of North Carolina, Chapel Hill, 1992 (Colloquium), 2004 (Colloquium).

44. University of Arizona, Tucson, 1992 (Colloquium).

45. California Institute of Technology, 1992.

46. Boston University, 1992, 1995, 2002.

47. Northeastern University, 1992, 1993, 1994, 1995, 1996, 1997, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005

48. Brown University, 1992 (Colloquium), 1994, 1997, 2001 (Colloquium).

49. Yale University, 1992 (Colloquium).

50. European Conference on Partial Differential Equations, Paris, 1992.

51. International Conference Index Theory–92, Oxford, England, 1992.

52. Mittag-Leffler Institute, Stockholm, 1992.

53. KTH, Stockholm, 1992 (Colloquium), 2001.

54. Linköping University, Sweden, 1992 (Colloquium), 2001 (Colloquium).

55. University Paris-6, 1993, 1995, 1999.

56. University of Bordeaux, 1993 (Colloquium).

57. University of Caen, 1993 (Colloquium).

58. Saint Jean de Monts, Conference on Partial Differential Equations, 1993.

59. Penn. State University, 1993 (Colloquium), 2002 (MASS Colloquium), 2002 (Colloquium).

60. Tel Aviv, Conference "Geometries in interaction", 1993.

61. Hebrew University, Jerusalem, 1993 (Colloquium).

62. Technion, Haifa, 1993 (Colloquium), 1998 (Colloquium).

63. Weizmann Institute, Rehovot, Israel, 1993 (Colloquium), 1998 (4 talks)

64. Harvard University, 1994, 1997.

65. University of Swansea, 1994.

66. University of Wales annual meeting, Greganog, Wales, 1994.

67. University of Cardiff, 1994.

68. Newton Institute for Mathematical Sciences, Cambridge, England, 1994.

69. Conference on Complex and Hypercomplex Analysis, Mexico, 1994.

70. University of Michigan, Ann Arbor, 1994.

71. Kings College, London, England, 1994.

72. University of Brighton, England, 1994.

73. Joint AMS - Israeli Math. Union meeting, Jerusalem, Israel, 1995.

74. Schrödinger Institute of Mathematical Physics, Vienna, Austria, 1995.

75. Vito Volterra Center, University Roma-2, Italy, 1995.

76. University of Auckland, New Zealand, 1995.
77. University of Melbourne, Australia, 1995 (Colloquium).
78. University of Adelaide, Australia, 1995 (2 lectures), 1996.
79. University of Sydney, Australia, 1995 (Colloquium).
80. Ben Gurion University, Ber Sheva, Israel, 1995, 1998.
81. Tel Aviv University, 1995, 1998
82. Imperial College, London, England, 1995.
83. University Paris-6, 1995, 1999.
84. Fields Institute, Waterloo, Canada, 1995.
85. Joint AMS-MAA meetings, Orlando, Florida, Special Session on "Geometry, Topology, and Analysis on Noncompact Manifolds", January 1996.
86. London Mathematical Society Symposium on Partial Differential Equations and Spectral Theory, Durham, England, July 1996 (Plenary talk).
87. Humboldt University, Berlin, Germany, 1996, 1998.
88. Wabash Extramural Modern Analysis Miniconference, Indianapolis, 1996 (Plenary talk).
89. Moscow State University, 1996, 1997, 2002.
90. Moscow Mathematical Society, 1996, 1997, 2002.
91. Joint AMS-MAA meetings, San Diego, California, January 1997 (Plenary talk).
92. University of Chicago, 1997 (Colloquium).
93. University of Miami, 1997 (Colloquium).
94. Georgia Institute of Technology, 1997.

95. Conference “Positive solutions of elliptic and parabolic equations”, Haifa, Israel, 1997.
96. Workshop “Symplectic geometry”, Fields Institute, Toronto, 1997.
97. Workshop in Geometry and Physics, Adelaide, Australia, 1997.
98. Special session “Geometric Analysis and Spectral Theory”, AMS meeting, Montreal, 1997.
99. Dartmouth College, 1998 (Colloquium).
100. UNAM (Mexico), 1998 (Colloquium), 2002 (Colloquium).
101. Bar Ilan University (Israel), 1998 (Colloquium).
102. University of Haifa, 1998.
103. International Workshop in Topology (Tel Aviv, 1998).
104. Conference on Geometric Analysis and Singular Spaces (Oberwolfach, 1998).
105. Potsdam University (Germany), 1998, 2002.
106. Conference “Differential geometry and applications”, Brno (Czech republic), 1998.
107. Conference “Functional analysis, partial differential equations and applications”, Rostock (Germany), 1998.
108. Minisymposium “Spectral invariants, heat equation approach”, Roskilde (Denmark), 1998.
109. Miniconference “Scattering and spectral theory”, Jerusalem, 1998.
110. Université Toulouse-I, 1999.
111. University of Missouri, Columbia, 1999 (Colloquium and 2 seminar talks).
112. Conference “Differential Geometrie im Großen”, Oberwolfach, 1989 and 1999.

113. Volkswagen Foundation Conference, Berlin, 1999.

114. University of Münster, 1999.

115. Workshop in Geometric Analysis, Sarasota (Florida), 2000.

116. York University, Toronto, Canada, 2000 (Colloquium).

117. AMS special session “Partial Differential Equations and Dynamical Systems”, Lowell (Massachusetts), 2000.

118. Euresco conference “Partial Differential Equations and their Applications to Geometry and Physics”, Castelvecchio Pascoli, Italy, 2000 (plenary talk).

119. Pacific Northwest Geometry Seminar, Portland, Oregon, 2000.

120. Texas A&M University, Fluid Dynamics seminar, 2001.

121. Rice University, 2001 (Colloquium)

122. Workshop “Quantization and non-commutative geometry”, MSRI, Berkeley, 2001

123. Workshop “Geometric Scattering Theory and Elliptic Theory on Noncompact and Singular Spaces”, MSRI, Berkeley, 2001.

124. University of California, Davis, Analysis/Mathematical Physics seminar, 2001.

125. International conference “Differential Equations and Related Topics”, dedicated to 100th annyversary of I.G.Petrovskii, Moscow State University, Moscow, Russia, 2001, 2004.

126. International conference “Topological Analysis of Manifolds and Submanifolds”, dedicated to J. Levine, Tel Aviv, Israel, 2001.

127. AMS special session “Spectral theory of Schrödinger operators”, Columbus, OH, 2001.

128. AMS special session “L^2-methods in algebraic and geometric topology”, Columbus, OH, 2001.

129. Mathematical Symposium “Partial Differential Equations” in Honor of Professor Mark Vishik, Freie Universität, Berlin, Germany, 2001.

130. International Conference “Fundamental Mathematics Today”, dedicated to the 10th anniversary of the Independent University of Moscow, Moscow, Russia, 2001.

131. Mini-conference on Spectral Geometry ad Related Topics (Indianapolis), 2002.

132. SFB Colloquium (Berlin), 2002.

133. Workshop “Geometric Methods in Physics” (Bialowieza, Poland), 2002, 2003, 2004.

134. Gdansk University (Poland), 2002 (Colloquium).

135. Workshop “Spectral theory of Schrödinger operators”, Mittag-Leffler Institute (Stockholm), 2002.

136. Independent University of Moscow, 2002 (Colloquium).

137. Montreal CRM/ISM Colloquium (4 universities Colloquium), 2003.

138. McGill University (Montreal), Analysis seminar, 2003.

139. International Conference in PDE and Their Applications (in celebration of 80th birthday by A.I. Volpert), Haifa (Israel).

140. Purdue University (West Lafayette, IN), 2003 (Colloquium).

141. Workshop “Inverse Spectral Geometry” (Dartmouth College), 2003.

142. Texas Geometry and Topology Conference, (Texas Christian University), 2004.

143. Seminar “Representations Theory and Related Topics”, (University of North Carolina, Chapel Hill), 2004.

144. Workshop “Semi-classical Theory of Eigenfunctions and Partial Differential Equations” (CIRM, Montreal), 2004.

145. Tohoku University (Sendai, Japan), Geometry Seminar, 2004.

146. University of Tokyo, Analysis seminar, 2004.

147. International Conference "Degenerate Partial Differential Equations and Singular Geometries" (Potsdam, Germany), 2004.

148. International Conference in Analysis and Geometry (Novosibirsk, Russia), 2004.

149. AMS Sectional Meeting, Northwestern University, 2004.

150. University of Connecticut, Storrs, 2004 (Colloquium).

151. University of Texas, Austin, Analysis Seminar, 2005.

152. University of Kentucky (Lexington), Colloquium, 2005.

153. Research Institute in Mathematical Sciences (Kyoto, Japan), Colloquium, 2005.

154. International Conference "Analysis and Geometry of Boundary Value Problems", Roskilde, Denmark, 2005.

155. International Conference "Differential Equations and their Applications" dedicated to Vladimir Kondratiev's 70th birthday, Samara (Russia), 2005.

156. Symposium "Operator Theory and Spectral Analysis", Durham (England), 2005.

157. Workshop "Model Reduction and Coarse-Graining for Multiscale Phenomena", Leicester (England), 2005.

8 RESEARCH AND VISITING PROFESSOR POSITIONS AFTER 1988

1. Institute des Hautes Etudes Scientifiques (1988-1989, 2 months; 1991, 1 week; 1993, 2 weeks; 1994, 6 weeks; 1995, 2 weeks; 1996, 2 weeks; 1999, 2 months).

2. University Paris-Nord (1989, 1 month).
3. Ohio State University, Columbus, OH (1991, 10 weeks; 1995, 1 week).
4. University of Maryland (1991, 2 weeks; 1992, 2 weeks; 1993, 2 weeks; 1994, 2 weeks).
5. Courant Institute of Mathematical Sciences, New York University (1991, 2 weeks; 1997, 1 week).
6. Eidgenossische Technische Hochschule, Zürich (1991, 1 month; 1992, 2 months; 1993, 1.5 months).
7. Massachusetts Institute of Technology (1991-1992, 1 year).
8. University of Toronto (1992, 1 month).
9. Institute Mittag-Leffler, Stockholm (1992, 1 month; 2002, 2 weeks).
10. University of Wales at Swansea and Cardiff (1994, 1 month).
11. Newton Institute for Mathematical Sciences, Cambridge, England (1994, 2 weeks).
12. University of Augsburg (1994, 1 month).
13. Vito Volterra Center, University Roma-2, Italy (1995, 1 week).
14. Stanford University (1995, 2 weeks).
15. Schrödinger Institute of Mathematical Physics, Vienna, Austria (1995, 1 month).
16. University of Adelaide, Australia (1995, 1 month; 1996, 1 month; 1997, 1 month).
17. Tel Aviv University, Israel (1995, 1 month; 1998, 1 month).
18. Fields Institute, Waterloo, Canada (1995, 5 weeks), 1997 (4 weeks).
19. Humboldt Universität, Berlin, Germany (1996, 2 weeks; 1998, 2 weeks; 2002, 2 weeks).

20. UNAM, Mexico (1998, 2 weeks).
21. The Weizmann Institute of Science, Rehovot, Israel (Meyerhoff professor, 1998, 3 months).
22. University of Münster (1999, 1.5 months).
23. Oberwolfach Mathematical Institute (1999, 1 month).
24. Oberwolfach Mathematical Institute (2001, 1 month).
25. MSRI, Berkeley (2001, 3 weeks).
26. Independent University of Moscow (2002, 5 weeks).
27. Zürich University (2003, 2 weeks)
28. Zürich University (2004, 3 weeks)
29. ETH, Zürich (2005, 2 weeks)

9 PROFESSIONAL MEMBERSHIPS

1. Moscow Mathematical Society (A member of the board in 1990-1992)
2. American Mathematical Society
3. International Association of Mathematical Physics

10 EDITORIAL BOARDS

1. Seminar Analysis of the Karl-Weierstrass-Institut, 1985 – 1991.
2. Potential Analysis, 1992 – .
3. Russian Journal of Mathematical Physics, 1993 – .
4. Research Monographs/Lecture Notes series, World Scientific, 1999 – .
5. Annals of Global Analysis and Geometry, 2001– .
6. Proceedings of the American Mathematical Society, 2005 – .

11 EDUCATIONAL ACTIVITIES

Organized Basic Notions Seminar in Mathematics Department of Northeastern University beginning Fall 1993.

Organized Mainly Analysis Seminar in Mathematics Department of Northeastern University beginning Fall 2005.

Participated in organization of Analysis & Geometry Seminar in Mathematics Department of Northeastern University beginning Fall 1992.

12 ACTIVITIES IN K-12 LEVEL TEACHING

During 1961-65 worked in mathematical facultative classes for High school students at the Department of Mechanics and Mathematics of Moscow State University

During 1965-66 and 1966-67 academic years worked in special mathematical classes of Moscow High school no. 2.

During 1967-69 worked in Correspondence Mathematical School at Moscow State University.

During 1972-1989 participated in different summer schools in Riga, Petrozavodsk and Krasnoyarsk (15 times total) for high school students selected by their abilities in mathematics and physics.

During 1986-89 worked as the scientific supervisor in the evening mathematical school for high school students at the Department of Mechanics and Mathematics of Moscow State University.

From 2002 a member of the Founding Board of Advanced Math and Science Academy Charter School. Member of the Board of Trustees from 2004.

From 2002 participated in development of a web site http://www.mathcircle.org containing a list of mathematical problems for high school students.

13 APPLIED WORK AND WORK WITH INDUSTRY

In Moscow during several periods did some applied work in Pattern Recognition and also in use of magnetic fields in some Control problems.

14 SERVICE

During 1990-1992 was in the elected board of Moscow Mathematical Society.

From 1991 participated in the organization of a joint Russian-German mathematical program sponsored by Volkswagen being a cochairman of the Partial Differential Equations Committee there.

In 1992 was in the Founding Board of the Independent University of Moscow.

In 1993 was in an AMS committee which assigned grants to mathematicians from former Soviet Union.

In 1994 was in the panel of the International Science Foundation (Soros foundation) for the former Soviet Union.

In 1995 was one of the organizers (together with R.McOwen and Ch.King) of the special session "Partial Differential Equations in Geometry and Mathematical Physics" in 903rd Meeting of the AMS (Northeastern University, Boston)

1995-1997 Graduate Director in Department of Mathematics of Northeastern University.

One of the organizers (together with L.Friedlander) of the special session "Spectral Geometry on Non-compact Manifolds" in the joint AMS-MAA annual meetings (San Diego, January 1997).

Member of the American Mathematical Society Eastern Section Program committee 1998–2000; Chair of this committee 1999–2000.

Member of the Scientific Committee of the EuroConference on Partial Differential Equations and their Applications to Geometry and Physics, Castelvecchio Pascoli, Italy, June 2000.

Member of College of Arts and Sciences Full Professors Promotion Advisory Committee (Northeastern University), 2001–2002.

One of the organizers of the XXI, XXII, XXIII and XXIV Workshops “Geometric Methods in Physics” in Bialowieza (Poland), June-July 2002, 2003, 2004, 2005.

One of the organizers (together with Maxim Braverman and Victor Nistor) of the special session “Elliptic Operators on Noncompact Manifolds” during AMS meeting in Northeastern University, October 5-6, 2002.

Member of AMS committee “Math in Moscow” from 2003. Chair of this committee from 2005.

Member of the Program Committee of the International Conference “Differential Equations and their Applications” dedicated to Vladimir Kondratiev’s 70th birthday, Samara (Russia), June 28 – July 2, 2005.

15 PUBLICATIONS

Remark. Items [13,24,44,53,64,87,88,92,119,126,134] in the list are books.

1. *Certain properties of generalized ω-limiting sets of dynamical systems* Vestnik Moskov. Univ. Ser. I Mat. Mekh. 21 (1966), no.3, 58-60, MR 33 #4912

2. *Uniqueness of solution of the Cauchy problem for convolution equations.* Mat Sb. (N.S.) 72 (114) (1967), 321-336, MR 35 #1987

3. *Factorization of matrix functions dependent on a parameter in normed rings and related questions in the theory of Noetherian operators.* Mat sb. (N.S.) 73 (115) (11967), 610-629, MR 36 #727

4. *Operators "in general position" in Hilbert spaces.* Mat. Zametki 1 (1967), 699-702, MR 35 #4750

5. *On the index of higher-dimensional Wiener–Hopf equations in a half–space.* Uspekhi Mat. Nauk 24 (1969), no.3 (147), 222, MR 40 #6312

6. *On holomorphic families of subspaces of a Banach space.* Mat. Issled. 5 (1970), vyp.4 (18), 153-165, MR 40 #2948a. English translation: Integral Equations and Operator Theory, 2/3 (1979), no. 3, 407-420) MR 80m: 46045

7. *Pseudo-differential operators in* $\mathbf{R}^n$. Dokl. Akad. Nauk SSSR 196 (1971), 316-319, MR 42 #8341. English translation: Soviet Math. Dokl., 12 (1971), no. 1, 147-151

8. *The local principle in the factorization problem.* Mat. Issled. 6 (1971), vyp. 1 (19), 174-180, MR 44 #2930

9. *Letter to the editors.* Mat. Issled. 6 (1971), vyp 1 (19), 180, MR 44 #29488

10. *On the index of families of Wiener–Hopf operators.* Mat. Sb. (N.S.) 84 (126) (1971), 537-558, MR 58 #18614. English translation: Math. USSR Sbornik, 13 (1971), no. 4, 529-551

11. *Factorization of matrices depending on a parameter and elliptic equations in a half-space.* Mat. Sb. (N.S.) 85 (127) (1971), 65-84, MR 30 #459. English translation: Math. USSR Sbornik, 14 (1971), no. 1, 65-84

12. (with F.A.Berezin) *Symbols of operators and quantization. Hilbert space operators and operator algebras.* (Proc. Intern. Conf., Tihany, 1970) pp. 21-52. Colloq. Math. Soc. Janos Bolyai, No. 5, North-Holland, Amsterdam, 1972, MR 51 #2529

13. (with F.A.Berezin) *Lectures on quantum mechanics.* Moscow State University Publishers, 1972, 294pp.

14. *Certain properties of pseudo-differential operators with nonsmooth symbols.* Dokl. Akad. Nauk SSSR 207 (1972), 551-553, MR 47 #2211

15. *Conditions for the discreteness of the spectrum of certain operators.* Mat. Zametki 11 (1972), 233-240, MR 45 #2211

16. (with V.N.Tulovskii) *The asymptotic distribution of the eigenvalues of pseudo-differential operators in* $\mathbf{R}^n$. Mat. Sb. (N.S.) 92 (134) (1973), 571-588, MR 48 #9465

17. (with V.N.Tulovskii) *The asymptotic behaviour of the eigenvalues of pseudo-differential operators in* $L^2(\mathbf{R}^n)$. Uspekhi Mat. Nauk 28 (1973) no.5 (173), 242, MR 52 #14688

18. *The spectral properties of operators with covariant and contravariant symbols and a certain variational principle.* Vestnik Moskov. Univ. Ser. I Mat. Mekh. 28 (1973), no. 3, 51-57, MR 50 #5534

19. *Spaces of almost periodic functions and differential operators.* Funktsional. Anal. i Prilozhen. 8 (1974), no. 4, 95-96, MR 56 #16365

20. *Differential and pseudo-differential operators in spaces of almost periodic functions.* Mat. Sb. (N.S.) 95 (137) (1974), 560-587, MR 50 #10911. English translation: Math. USSR Sbornik, 24 (1974), no. 4, 547-573

21. (with L.A.Bagirov) *The stabilization of the solution of the Cauchy problem for parabolic equations with coefficients that are almost periodic in the space variables.* Differencial'nye Uravnenija 11 (1975), no.12, 2205-2209, MR 54 #10816

22. *The Favard–Muhamadiev theory and pseudo-differential operators.* Dokl. Akad. Nauk SSSR, 225 (1975), no.6, 1278-1280, MR 53 #6364. English translation: Soviet Math. Dokl., 16 (1975), no. 6, 1646-1649

23. *Elliptic almost periodic operators and von Neumann algebras.* Functional. Anal. i Prilozhen. 9 (1975), no. 1, 89-90, MR 58 #23168

24. *Problems of mathematical olympiads for students.* Moscow State University Publishers, 1975, 48pp.

25. (with V.I.Arnold, A.A.Kirillov, V.M.Tikhomirov) *On the first all-union mathematical olympiad for students.* Uspekhi Mat. Nauk, 30 (1975), no. 4, 281-288

26. *The density of states for elliptic operators with almost periodic coefficients.* Conference on Differential equations and Applications (Ruse, 1975). Godisnik Viss. Ucebn. Zaved. Prilozhna Mat. 11 (1975), no.2, 209-216 (1977), MR 57 #13245n

27. *The coincidence of the ordinary and the almost periodic spectrum of an elliptic operator.* Uspekhi Mat. Nauk 30 (1975), no. 3, (183), 185-186, MR 53 #6358

28. *The essential selfadjointness of uniformly hypoelliptic operators.* Vestnik Moskov. Univ. Ser. I Mat. Mekh. 30 (1975), no. 2, 91-94, MR 52 #14598

29. *The regularity of generalized almost periodic solutions of hypoelliptic equations.* Vestnik Moskov. Univ. Ser. I Mat. Mekh. 30 (1975), no. 5, 53-57, MR 52 #14582

30. *Almost periodic elliptic operators and von Neumann algebras.* Voronezh Gos. Univ. Trudy Naucn.-Issled. Inst. Mat. VGU Vyp. 17, Teor. Operator. Uravnenii (1975), 95-99, MR 58 #1738

31. (with I.M.Gelfand, Ju.I.Manin) *Poisson brackets and the kernel of the variational derivative in the formal calculus of variations.* Funktsional. Anal. i Prilozhen. 10 (1976), no.4, 30-34, MR 55 #13486

32. *Theorems on the coincidence of the spectra of a pseudo-differential almost periodic operator in the spaces $L^2(\mathbf{R}^n)$ and $B^2(\mathbf{R}^n)$.* Sibirsk. Mat. Zh. 17 (1976), no. 1, 200-215, MR 53 #6365. English translation: Plenum Publishing Corporation, 1976, 158-170

33. *Pseudo-differential almost periodic operators and von Neumann algebras.* Trudy Moskov. Mat. Obsc. 35 (1976), 103-164, MR 58 #30521. English translation: Trans. Moscow Math. Soc., 1979, Issue 1, 103-166

34. *Weyl's theorem for the Schrödinger operator with an almost periodic potential.* Vestnik Moskov. Univ. Ser I Mat. Mekh. 31 (1976), no. 2, 84-88, MR 53 #13868

35. (with B.V.Fedosov) *The index of random elliptic operators and of families of them.* Dokl Akad. Nauk SSSR, 236 (1977), no. 4, 812-815, MR 58 #24397

36. *Spectral theory of elliptic operators with almost periodic coefficients.* Trudy Sem. of S.L.Sobolev, no.2 (1977), 170-172

37. (with I.M.Gelfand and Yu.I.Manin) *Nonlinear partial differential equations and Hamiltonian structures.* Uspekhi Mat. Nauk, 32 (1977), no. 3, 209-210

38. (with A.I.Gusev) *Elliptic operators with random coefficients.* In the book "Limit theorems for stochastic processes", Institute of Math. Ukrainean Acad. of Sci., 1977, 98-107

39. (with B.V.Fedosov) *The index of random elliptic operators. I.* Mat. Sb. (N.S.) 106 (148) (1978), no. 1, 108-140, MR 58 #18610. English translation: Math. USSR Sbornik, 34 (1978), no. 5, 671-699

40. (with B.V.Fedosov) *The index of random elliptic operators. II.* Mat. Sb. (N.S.) 106 (148) (1978), no. 3, 455-483, MR 58 #18611. English translation: Math. USSR Sbornik, 35 (1979), no. 1, 131-156

41. (with B.V.Fedosov) *The index of random elliptic operators and the oblique derivative problem.* Uspekhi Mat. Nauk, 33 (1978), no. 3, 154

42. *The density of states of self-adjoint elliptic operators with almost periodic coefficients.* Trudy Sem. Petrovsk. No. 3 (1978), 243-275, MR 58 #17587. English translation: Amer. Math. Soc. Transl. (2), vol. 118 (1982), 307-339

43. *Almost periodic functions and partial differential operators.* Uspekhi Mat. Nauk 33 (1978), no. 2 (200), 3-47, MR 58 #30522. English translation: Russian Math. Surveys, 33 (1978), no. 2, 1-52

44. *Pseudo-differential operators and spectral theory.* "Nauka", Moscow, 1978, 279pp. MR 80h: 47057. English translation: Springer-Verlag, 1987, second edition 2001

45. *Almost periodic elliptic operators.* Proc. of an All-union Conference on Partial Differential Equations (Moscow State Univ., Moscow, 1976) (Russian), pp. 245-248, Moskov. Gos. Univ., Mekh.-Mat. Facul'tet, Moscow, 1978 (See MR 80f: 35002)

46. (with P.E.Dedik) *Elliptic operators in spaces of homogeneous random fields and the stabilization of solutions of parabolic equations with random coefficients.* Dokl. Akad. Nauk SSSR, 249 (1979), no. 5, 1043-1046

47. *The spectral theory and the index of elliptic operators with almost periodic coefficients.* Uspekhi Mat. Nauk 34 (1979), no. 2, (206), 95-135, MR 81f: 35090. English translation: Russian Math. Surveys, 34 (1979), no. 2, 109-157

48. *Local Favard theory.* Vestnik Moskov. Univ. Ser I Mat. Mekh. No. 2 (1979), 31-36, MR 81i:35015

49. (with P.E.Dedik) *Random pseudo-differential operators and stabilization of the solutions of parabolic equations with random coefficients.* Mat. Sb. (N.S.) 113 (155) (1980), no. 1 (9), 41-64, MR 82f: 35190. English translation: Math. USSR Sbornik, 41 (1982), no. 1, 33-52

50. *The spectrum and the distribution function for a transversally elliptic operator.* Funktsional. Anal. i Prilozhen. 15 (1981), no. 1, 90-91, MR 82h: 58050

51. (with N.N.Bogolyubov, I.M.Gelfand, R.L.Dobrushin, A.A.Kirillov, M.G.Krein, D.A.Leites, R.A.Minlos, Ya.G.Sinai) *Felix Alexandrovich Berezin (obituary)* Uspekhi Mat. Nauk, 36, no.4 (1981), 185-190

52. (with A.V.Brenner) *The Atiyah–Bott–Lefschetz theorem for manifolds with boundary.* Functional. Anal. i Prilozhen. 15 (1981), no. 4, 67-68

53. (with L.A.Basova and L.A.Epstein) *Lectures and problems in mathematics.* Prosveschenie Publishers, 1981, 96pp.

54. (with V.Ya.Ivrii) *On the asymptotic behaviour of the spectral shift function.* Dokl. Akad. Nauk SSSR, 263 (1982), no. 2, 283-284, Errata:265 (1982), no. 5, 1032, MR 83k: 58083a,b

55. *Topology and the lay of the land (mathematical topography).* Quant, no. 8 (1982), 10-15. English translation: Quantum, Sep/Oct92 (Feature)

56. (with I.N.Bondareva) *Increasing asymptotic solution of the Korteweg – de Vries equation and of its higher analogues.* Dokl. Akad. Nauk SSSR, 267 (1982), no. 5, 1035-1038, MR 84k: 35121. English translation: Soviet Math. Dokl., 26 (1982), no. 3, 716-719

57. (with S.M.Kozlov) *A theorem on the coincidence of spectra for random elliptic operators.* Functsional. Anal. i Prilozhen. 16 (1982), no. 4, 74-75, MR 84d: 35112

58. *Spectral properties and distribution function of the spectrum of a transversally elliptic operator.* Trudy Sem. Petrovsk., No. 8 (1982), 239-258, MR 83m: 58073

59. (with S.M.Kozlov) *On the structure of functions belonging to the class S of self-adjoint elliptic operators in $\mathbf{R}^n$.* Uspekhi Mat. Nauk 37 (1982), no. 2 (224), 201-202, MR 84h: 35175

60. (with M.S.Agranovich, I.M.Gelfand, Yu.A.Dubinskii, O.A.Oleinik, S.L.Sobolev) *Mark Iosifovich Vishik (on his 60th anniversary)* Uspekhi Mat. Nauk 37 (1982), no. 4 (226), 213-220

61. (with T.E.Bogorodskaya) *Variational principle for the density of states of random pseudo-differential operators and its applications.* Funktsional. Anal. i Prilozhen. 17 (1983), no. 2, 66-67, MR 84m: 35126

62. (with Georgi St. Popov) *Complete asymptotic expansion of the spectral function for second-order elliptic operators in $\mathbf{R}^n$.* Uspekhi Mat. Nauk 38 (1983), no. 1 (229), 187-188, MR 84f: 35111

63. *Essentiality of the spectrum of pseudo-differential operators in the Besikovich space $B^2(\mathbf{R}^n)$.* Vestnik Moskov. Univ. Ser I Mat. Mekh. 1982, no. 5, 56-59, MR 85b: 47056

64. (with F.A.Berezin) *The Schrödinger equation.* Moskov. Gos. Univ., Moscow, 1983, 392pp. (in Russian) MR 85m: 81001. Enlarged English translation: Kluwer Academic Publishers, Dordrecht e.a., 1991, xi + 555pp.

65. (with Georgi St. Popov) *Asymptotic expansion of the spectral function for second-order elliptic operators in* $\mathbf{R}^n$. Funktsional. Anal. i Prilozhen. 17 (1983), no. 3, 37-45, MR 85b: 35041

66. (with S.M.Kozlov) *Coincidence of spectra of random elliptic operators.* Mat. Sb. (N.S.) 123 (165) (1984), no. 4, 460-476, MR 85i: 35149. English translation: Math. USSR Sbornik, 51 (1985), no. 2, 455-471

67. (with A.K.Zvonkin) *Nonstandard analysis and singular perturbations of ordinary differential equations.* Uspekhi Mat. Nauk 39 (1984), no. 2 (236), 77-127, MR 85j: 34119

68. (with S.A.Smagin) *The zeta-function of a transversally elliptic operator.* Uspekhi Mat. Nauk 39 (1984), no 2. (236), 187-188, MR 85j: 58146

69. *Pseudo-difference operators and their inversion.* Dokl. Akad. Nauk SSSR, 276 (1984), no. 3, 567-570, MR 86i: 47041. English translation: Soviet Math. Dokl., 29 (1984), no. 3, 574-577

70. (with G.A.Meladze) *Algebras of pseudo-differential operators on unimodular Lie groups.* Dokl. Akad. Nauk SSSR, 279 (1984), no.3, 542-545, MR 86f: 58153. English translation: Soviet Math. Dokl., 30 (1984), no. 3, 689-692

71. (with S.A.Smagin) *The zeta-function of a transversally elliptic operator.* Sibirsk. Mat. Zh., 25 (1984), no. 6, 158-166, MR 86d: 58119

72. *Pseudo-difference operators and their Green function.* Izv. Akad. Nauk SSSR Ser. Mat. 49 (1985), no. 3, 652-671, MR 87b: 39005. English translation: Math. USSR Izvestiya, 26 (1986), no. 3, 605-622

73. (with Dieter Schenk) *Asymptotic expansion of the density of states and the spectral function of the Hill operator.* Mat. Sb. (N.S.) 128 (170)

(1985), no. 4, 474-491, MR 87h: 34078. English translation: Math. USSR Sbornik, 56 (1987), no. 2, 473-490

74. (with G.A.Meladze) *Uniform pseudo-differential operators on unimodular Lie groups.* Soobshch. Akad. Nauk Gruzin. SSR 118 (1985), no. 1, 21-24, MR 87e: 58196

75. (with I.N.Bondareva) *Uniqueness of the solution of the Cauchy problem for the Korteweg – de Vries equation in classes of increasing functions.* Vestnik Moskov. Univ. Ser. I Mat. Mekh. 1985, no. 3, 35-38, MR 87c: 35133. English translation: Moscow University Mathematics Bulletin, 40 (1985), no. 3, 53-57

76. (with Dieter Schenk) *Asymptotic expansion of the spectral function of the Hill operator.* Funktsional. Anal. i Prilozhen. 20 (1986), no. 1, 89-90, MR 87k: 34092

77. (with G.A.Meladze) *Properly supported uniform pseudo-differential operators on unimodular Lie groups.* Trudy Sem. Petrovsk., No. 11 (1986), 74-97, MR 87i: 58166

78. (with T.E.Bogorodskaya) *The variational principle and asymptotic behaviour of the density of states for random pseudo-differential operators.* Trudy Sem. Petrovsk., No.11 (1986), 98-117, MR 87j: 35389

79. (with S.P.Novikov) *Morse inequalities and von Neumann* II_1*-factors.* Dokl. Akad. Nauk SSSR, 289 (1986), no. 2, 289-292, MR 88i: 58065. English translation: Soviet Math. Dokl., 34 (1987), no. 1, 79-82

80. (with S.P.Novikov) *Morse inequalities and von Neumann algebras.* Uspekhi Mat. Nauk 41 (1986), no. 4, 163-164

81. (with S.P.Novikov) *Morse theory and von Neumann invariants of non-simply connected manifolds.* Uspekhi Mat. Nauk 41 (1986), no. 5, 222-223

82. *Spectral theory of random elliptic operators.* Proc of the 1st World Congress of the Bernoulli Society of Math. Statistics and Probability Theory, v.II (1986), 729

83. (with I.N.Bondareva) *Equations of the Korteweg – de Vries type in classes of increasing functions.* Uspekhi Mat. Nauk 42 (1987), no. 4, 144

84. (with G.A.Meladze) *Functional calculus of pseudo-differential operators on unimodular Lie groups.* Trudy Sem. Petrovsk., No. 12 (1987), 164-200

85. (with D.V.Efremov) *Asymptotics of the distribution function of the spectrum for automorphic operators of Schrödinger type on the hyperbolic space.* Uspekhi Mat. Nauk 43 (1988), no. 4, 164-165

86. (with D.V.Efremov) *Spectrum distribution function and variational principle for automorphic operators on the hyperbolic space.* Seminaire Equations aux derivees partielles, 1988-1989, Ecole Polytechnique, Expose n. VIII.

87. (with Yu.V.Egorov) *Linear partial differential equations. Foundations of classical theory.* Encyclopaedia of Mathematical Sciences, vol. 30, Partial Differential Equations, I, 1988, VINITI, 1-262. English translation: Springer-Verlag, Berlin e.a., 1992, 259pp.

88. (with Yu.V.Egorov) *Linear partial differential equations. Elements of modern theory.* Encyclopaedia of Mathematical Sciences, vol. 31, 1988, VINITI, 5-125. English translation: Springer-Verlag, Berlin e.a., 1994.

89. *Mathematical analysis for solving of physical problems.* In the book: "At the junction of all sciences". Krasnoyarsk University Press, 1989, 124-177

90. (with V.I.Arnold, M.I.Vishik, Ju.S.Iliashenko, A.S. Kalashnikov, V.A.Kondratiev, S.N.Kruzhkov, E.M.Landis, V.M.Millionschikov, O.A.Oleinik, A.F.Filippov) *Some unsolved problems in differential equations and mathematical physics.* Uspekhi Mat. Nauk 44 (1989), no. 4, 191-202

91. (with I.N.Bondareva) *Equations of Korteweg – de Vries type in classes of increasing functions.* Trudy Sem. Petrovsk., No.14 (1989), 45-56.

English translation: Journal of Soviet Mathematics, 51 (1990), no. 3, 2323-2332

92. (with G.V.Rosenblum and M.Z.Solomiak) *Spectral theory of differential operators.* Encyclopaedia of Mathematical Sciences, vol. 64 (1989), VINITI, 1-242. English translation: Springer-Verlag, Berlin e.a., 1994

93. (with S.Seifarth) *A Lefschetz fixed point formula for manifolds with cylindric ends.* Annals of Global Analysis and Geometry, 9, no.2 (1991), 99-108

94. (with S.Seifarth) *A Lefschetz fixed point formula on manifolds with cylindrical ends.* C. R. Acad. sci., Paris, t.310, Ser. I (1990), 849-853

95. *Weak Bloch property and weight estimates for elliptic operators.* Seminaire Equations aux derivees partielles, 1989-1990, Ecole Polytechnique, Exposé n. V., V-1-V-20.

96. (with J.Sjöstrand) *On the equality between weak and strong extensions.* Seminaire Equations aux derivees partielles, 1989-1990, Ecole Polytechnique, Appendix a l'Exposé n. V, V-1-V-10

97. *A Lefschetz fixed point formula in reduced L^2-cohomologies on manifolds with cylinders.* Symposium "Analysis on Manifolds with Singularities", Breitenbrunn 1990; Teubner-Texte zur Mathematik, Band 131 (1992), 290-297

98. (with A.V.Brenner) *Atiyah-Bott-Lefschetz formula for elliptic complexes on manifolds with boundary.* Sovremennye Problemy Matematiki, Noveishie Dostizheniya (Modern Problems in Mathematics, Newest Achievements), v.38 (1990), VINITI, 119-183

99. (with F.Palmeira) *Pavel Samuilovich Urysohn.* The Mathematical Intelligencer, vol.12 (1990), no.4, 39

100. *Eigenfunctions and spectra of elliptic operators on manifolds of bounded geometry.* Uspekhi Matem. Nauk, 45(1990), no.4, 131

101. (with M.Gromov) *Near-cohomology of Hilbert complexes.* Uspekhi Matem. Nauk, 46(1991), no.6, 159-160

102. (with M.Gromov) *Von Neumann spectra near zero.* Geometric Analysis and Functional Analysis, 1 (1991), no. 4, 375-404

103. (with D.V.Efremov) *Spectral asymptotics for elliptic operators of Schrödinger type on hyperbolic space.* Trudy Sem. Petrovsk., no. 15 (1991), 3-32

104. (with M.Gromov) *Near-cohomology of Hilbert complexes and topology of non-simply connected manifolds.* Astérisque, 210 (1992), 283-294

105. (with M.Gromov) *The Riemann-Roch theorem for general elliptic operators.* C. R. Acad. Sci. Paris, t.314, Ser.I (1992), 363-367

106. *Spectral theory of elliptic operators on non-compact manifolds.* Astérisque, 207 (1992), 35-108

107. *Algebraic remarks on idempotent semirings and the Kernel Theorem in spaces of bounded functions.* Advances in Soviet Mathematics, vol. 13, American Math. Soc., Providence, R.I., 1992, 151-166

108. (with M.Gromov) *The Riemann–Roch theorem for general elliptic operators.* In: I.M.Gelfand Seminar, Part 1, Advances in Soviet Mathematics, vol. 16, Part 1, American Math. Soc., Providence, R.I., 1993, 211-241

109. (with M.Gromov) *The Riemann-Roch theorem for elliptic operators and solvability of elliptic equations with additional conditions on compact subsets.* Inventiones math., 117, no.1(1994), 165-180

110. *Discrete magnetic Laplacian.* Commun. Math. Phys., 164 (1994), 259-275

111. L^2 *Riemann-Roch theorem for elliptic operators.* Geometric and Functional Analysis, 5, no.2 (1995), 482-527

112. *Novikov inequalities for vector fields.* In: The Gelfand Mathematical Seminars 1993-1995 (I.M.Gelfand, J.Lepowsky, M.Smirnov eds.), Birkhäuser, 1996, 243-274

113. *Semiclassical asymptotics on covering manifolds and Morse inequalities.* Geometrical and Functional Analysis, 6, no.2 (1996), 370-409

114. *Remarks on the topology of the Hilbert grassmanian.* In: Contemporary Mathematical Physics. F.A. Berezin Memorial Volume. (R.L.Dobrushin, R.A.Minlos, M.A.Shubin, A.M.Vershik eds.) Advances in Math. Sciences 31, AMS Translations, Ser.2, vol.175, Providence, RI, 1996, 191-198

115. (with V.P.Maslov, A.M.Vershik, N.D.Vvedenskaya) *Alik Berezin in the recollections of friends .* In: Contemporary Mathematical Physics. F.A. Berezin Memorial Volume. (R.L.Dobrushin, R.A.Minlos, M.A.Shubin, A.M.Vershik eds.) Advances in Math. Sciences 31, AMS Translations, Ser.2, vol.175, Providence, RI, 1996, 191-198

116. (with M.S.Agranovich, A.V.Babin, L.R.Volevich, A.Yu.Goritskii, A.S.Demidov, Yu.A.Dubinskii, A.I.Komech, M.L.Krasnov, S.B.Kuksin, G.I.Makarenko, V.P.Maslov, V.M.Tikhomirov, A.V.Fursikov, V.V.Chepyzhov, A.I.Shnirelman) *Mark Iosifovich Vishik (on his 75th anniversary).* Russian Journal of Mathematical Physics, 4, no.4 (1996), 411-416

117. (with V.Mathai) *Twisted L^2 invariants of non-simply connected manifolds and asymptotic L^2 Morse inequalities.* Russian Journal of Mathematical Physics, 4, no.4 (1996), 499-526

118. (with M.S.Agranovich, A.V.Babin, L.R.Volevich, A.Yu.Goritskii, A.S.Demidov, Yu.A.Dubinskii, A.I.Komech, M.L.Krasnov, S.B.Kuksin, G.I.Makarenko, V.P.Maslov, V.M.Tikhomirov, A.V.Fursikov, V.V.Chepyzhov, A.I.Shnirelman) *Mark Iosifovich Vishik (on his 75th anniversary).* Uspekhi Mat. Nauk, 52, no.4 (1997), 225-232

119. (with Yu.V.Egorov) *Foundations of the classical theory of partial differential equations.* Springer-Verlag, Berlin e.a., 1998

120. (with M.Gromov, G.Henkin) *L^2 holomorphic functions on pseudo-convex coverings.* Contemporary Mathematics, 212, "Operator Theory for Complex and Hypercomplex Analysis". Proceedings of the Conference in Mexico City, December 12–17, 1994; E.Ramírez de Arellano, N.Salinas, M.V.Shapiro, N.L.Vasilevski eds., AMS, 1998, 81-94

121. *De Rham theory for extended L^2-cohomology* In: Voronezh Winter Mathematical Schools. Dedicated to Selim Krein. Amer. Math. Soc. Translations, Ser.2, vol.184. Advances in Math. Sciences, 37, Amer. Math. Soc., Providence, RI 1998, 217-232

122. (with M.Gromov, G.Henkin) *Holomorphic L^2 functions on coverings of pseudo-convex manifolds.* Geometric and Functional Analysis, vol.8, no.3 (1998), 552-585

123. (with I.M.Gelfand, V.Retakh) *Fedosov maniflods.* Advances in Math., 136, no.1 (1998), 104-140

124. *Elliptic boundary problems with relaxed conditions.* Amer. Math. Soc. Transl. Ser. 2, vol. 189 (1999), 219-231

125. *Classical and quantum completeness for the Schrödinger operators on non-compact manifolds.* In: B. Booss-Bavnbek, K. Wojciechowski (eds.), Geometric Aspects of Partial Differential Equations. Proceedings of a Minisymposium on Spectral Invariants, Heat Equation Approach, September 18-19, 1998, Roskilde, Denmark. Amer. Math. Soc., Providence, R.I., 1999, 257-271

126. (with Yu.V.Egorov, A.I.Komech) *Elements of the modern theory of partial differential equations* Springer-Verlag, Berlin e.a., 1999

127. (with V.A.Kondrat'ev) *Discreteness of spectrum for the Schrödinger operators on manifolds.* Functional Analysis and its Applications, 33, no.3 (1999), 85-87

128. (with V.A.Kondrat'ev) *Discreteness of spectrum for the Schrödinger operators on manifolds of bounded geometry.* Operator Theory: Advances and Applications, vol. 110 (dedicated to V.G.Maz'ya 60th anniversary), Birkhäuser Verlag, Basel, 1999, 185-226

129. *A sequence of connections and a characterization of Kähler manifodls.* In: M. Farber, W. Lück, S. Weinberger (eds.), Tel Aviv Topology Conference: Rothenberg Festschrift. International Conference on Topology June 1-5, 1998, Tel Aviv. Contemporary Math., Amer. Math. Soc., vol. 231 (1999), 265-270

130. *Spectral theory of the Schrödinger operators on non-compact manifolds.* In: B. Davies, Yu. Safarov (eds.), Spectral Theory and Geometry. ICMS Instructional Conference, Edinburgh 1998. London Math. Soc. Lecture Notes Series, vol. 273, Cambridge Univ. Press, 1999, 226-283

131. *Essential self-adjointness for magnetic Schrödinger operators on non-compact manifolds.* Seminaire Equations aux Dérivées Partielles, Ecole Polytechnique, 1998-1999, Exposé n^o XV, 9 Février 1999, XV-1 – XV-22

132. *Essential self-adjointness for semi-bounded magnetic Schrödinger operators on manifodls.* J. Funct. Anal., 186 (2001), no. 1, 92–116

133. (with B.A.Amosov, M.S.Birman, M.I.Vishik, L.R.Volevich, I.M.Gelfand, L.F.Friedlander) *Mikhail Semenovich Agranovich (to his 70th birthday).* Russian Math. Surveys, **56** (2001), no. 4, 163–168 (in Russian)

134. *Lectures on Equations of Mathematical Physics.* Modern Lecture Courses series. Moscow Center of Continuous Mathematical Education Publishers, Moscow, 2001, 303 pages (in Russian). Second Edition 2003

135. (with V.A.Kondrat'ev) *Discreteness of spectrum for the magnetic Schrödinger operators.* Communications in Partial Differential Equations, **27** (2002), 477–525

136. (with V.Mathai) *Semiclassical asymptotics and gaps in the spectra of magnetic Schroedinger operators.* Geometriae Dedicata, **91** (2002), 155–173

137. (with M.Braverman and O.Milatovich) *Essential self-adjointness of Schrödinger type operators on manifolds.* Russian Math. Surveys, **57**:4 (2002), 641–692

138. *List of selected talks by M. I. Vishik's seminar in Moscow.* In: Partial Differential Equations. Mark Vishik's Seminar. (M. S. Agranovich, M. A. Shubin editors.) American Math. Society. Advances in the Mathematical Sciences, **51**. AMS Translations Series 2, **206**. Providence, RI, 2002, 255–278

139. (with M.S.Agranovich, Yu.D.Burago, V.P.Khavin, V.A.Kondratiev, V.P.Maslov, S.M.Nikol'skii, Yu.G.Reshetnyak, B.R.Vainberg, M.I.Vishik, L.R.Volevich) *Vladimir G. Maz'ya. On the Occasion of His 65th Anniversary.* Russian J. of Math. Physics, **10** (2003), 239–244

140. (with V.Kondratiev and V.Maz'ya) *Discreteness of spectrum and strict positivity criteria for magnetic Schrödinger operators.* Commun. Partial Diff. Equations, **29** (2004), 489–521

141. (with Yu.Kordyukov and V.Mathai) *Equivalence of spectral projections in semiclassical limit and a vanishing theorem for higher traces in K-theory.* Journal für die reine und angewandte Mathematik (Crelle's Journal), **581** (2005), 193–236

142. (with V.Maz'ya) *Discreteness of spectrum and positivity criteria for Schrödinger operators.* Annals of Mathematics, **162** (2005), 919–942

143. (with T.Kappeler, P.Perry and P.Topalov) *The Miura map on the line.* International Mathematical Research Notices, **50** (2005), 3091–3133

144. (with V.Maz'ya) *Can one see the fundamental frequency of a drum?* Letters in Mathematical Physics, **74** (2005), 135–151

145. (with T.Sunada) *Geometric theory of lattice vibrations and specific heat.* Preprint math-ph/0512088 (31 pages). To appear in Pure and Applied Mathematics Quarterly.

146. (with T.Kappeler, P.Perry, P.Topalov) *Solutions of mKdV in classes of functions unbounded at infinity.* In preparation.

147. (with V.Kondratiev and V.Maz'ya) *Gauge optimization and spectral properties of magnetic Schrödinger operators.* In preparation.

148. *Partial differential equations.* Book in preparation.

149. *Von Neumann algebras and L^2 techniques in geometry and topology.* Book in preparation.

16 PUBLICATIONS EDITED

1. (with Yu.V.Egorov) *Partial Differential Equations I. Foundations of the Classical Theory.* Encyclopaedia of Mathematical Sciences, vol.30, Springer-Verlag, 1992

2. (with Yu.V.Egorov) *Partial Differential Equations II. Elements of the Modern Theory. Equations with Constant Coefficients.* Encyclopaedia of Mathematical Sciences, vol.31, Springer-Verlag, 1994

3. (with Yu.V.Egorov) *Partial Differential Equations III. The Cauchy Problem. Qualitative Theory of Partial Differential Equations.* Encyclopaedia of Mathematical Sciences, vol.32, Springer-Verlag, 1991

4. (with Yu.V.Egorov) *Partial Differential Equations IV. Microlocal Analysis and Hyperbolic Equations.* Encyclopaedia of Mathematical Sciences, vol.33, Springer-Verlag, 1993

5. (with Yu.V.Egorov) *Partial Differential Equations VI. Elliptic and Parabolic Operators.* Encyclopaedia of Mathematical Sciences, vol.63, Springer-Verlag, 1994

6. *Partial Differential Equations VII. Spectral Theory of Differential Operators.* Encyclopaedia of Mathematical Sciences, vol.64, Springer-Verlag, 1994

7. *Partial Differential Equations VIII. Overdetermined Systems. Dissipative Singular Schrödinger Operator. Index Theory.* Encyclopaedia of Mathematical Sciences, vol.65, Springer-Verlag, 1996

8. (with R.L.Dobrushin, R.A.Minlos, A.M.Vershik) *Contemporary Mathematical Physics. F.A.Berezin Memorial Volume.* Advances in the Mathematical Sciences 31. American Mathematical Society Translations, ser.2, vol.175. American Mathematical Society, 1996

9. (with R.L.Dobrushin, R.A.Minlos, A.M.Vershik) *Topics in Statistical and Theoretical Physics. F.A.Berezin Memorial Volume.* Advances in the Mathematical Sciences 32. American Mathematical Society Translations, ser.2, vol.177. American Mathematical Society, 1996

10. (with M.S.Agranovich, Yu.V.Egorov) *Partial Differential Equations IX. Elliptic Boundary Value Problems.* Encyclopaedia of Mathematical Sciences, vol.79, Springer-Verlag, 1997

11. (with M.S.Agranovich) *Partial Differential Equations. Mark Vishik's Seminar.* American Math. Society. Advances in the Mathematical Sciences, vol. 51. AMS Trtanslations Series 2, vol. 206. Providence, RI, 2002. VIII+278 pages.

12. (with M.Kontsevich, N.Nekrasov, A.Schwarz, D.Sternheimer) A special volume of Letters in Mathematical Physics dedicated to the memory of F. A. Berezin. Letters in Mathematical Physics, **74** (2005), issues 1,2,3.

17 BRIEF SUMMARY OF RESULTS

1. **Convolution equations** [2].
Uniqueness classes of solutions for the Cauchy problem were obtained for evolution equations containing convolutions with homogeneous distributions and their sums with respect to space variables. The obtained uniqueness classes were optimal in a natural scale of spaces of functions with a power growth at infinity.

2. **Factorization of matrix–functions and Wiener–Hopf equations** [3,5,8,10,11].
It was proved that for matrix-functions with natural smoothness conditions on the unit circle depending smoothly of additional parameters a smooth triangular factorization is possible with the same smoothness properties as for the matrix considered, locally with respect to the parameters. These results were used to construct explicitly solution of elliptic boundary value problems for matrix convolution equations of Vishik–Eskin type. The construction of non-trivial examples of matrix-functions with variable partial indices was done by means of a general K-theory index formula of families of one-dimensional Wiener–Hopf operators (such a formula was independently obtained by M.Atiyah and A.S.Dynin). A local principle for the factorization was obtained which

claims that the canonical factorization of a matrix-function exists if and only if local factorizations exist near every point on the circle.

3. **Holomorphic families of subspaces of Banach spaces** [6].
   It was proved that if a short exact sequence of holomorphic Banach vector bundles over a Stein space splits at every point then a global holomorphic splitting exists. The corollaries are the existence of global holomorphic left or right inverses and parametrices in case when the corresponding objects exist at every point and the parameter space is a Stein space.

4. **Pseudo-differential operators, quantization and symbols** [7,12,14,15,18,43].
   An algebra of global pseudo-differential operators in $\mathbf{R}^n$ with small Planck parameter was constructed in [7]. Different kinds of symbols were used to obtain conditions of discreteness of spectrum, boundedness of some operators with non-smooth symbols and inequalities of Feynman type for eigenvalues in [12,14,15,18].

   For pseudo-differential operators from uniform Hörmander classes the question about the structure of the inverse operator (if it exists in the usual Sobolev scale) naturally arose from some problems in almost periodic operators in [33]. It was proved in [43] that the inverse operator also belongs to the corresponding uniform Hörmander class. The main idea was to use weight spaces with power weight with a small parameter. The same result was also independently obtained by R.Beals but by entirely different method of characterization of uniform pseudo-differential operators by commutator estimates.

5. **Method of approximate spectral projection** [16,17,34,44,61,78].
   The method was suggested in a joint paper with V.Tulovskii [16,17] and was further developed by different authors (including V.Feigin, V.Roitburd, L.Hörmander, V.Beziaev, V.Levendorskii and others) into an universal method of investigation of asymptotics of eigenvalues of discrete spectrum. The main idea is that if we take an operator which has a symbol approximately equal to the indicator function of a set of lower values of the symbol of the operator under consideration then we shall obtain the operator the trace of which will be asymptotically equal

to the number of eigenvalues lying lower than we chosen level. In [16,17] this method was applied to obtain for the first time the asymptotics of the distribution function of eigenvalues for a class of global operators in $\mathbf{R}^n$ with an estimate of remainder. In [34,61,78] this method was applied to obtain the asymptotics of integrated density of states for some classes of almost periodic and random elliptic and hypoelliptic operators.

6. **Essential self-adjointness and coincidence of minimal and maximal extensions** [7,20,28,32,44,46,49,70,77,96,106,125,131,132, 137]. The idea first used in [7] was that regularity theorem for hypoelliptic equations may be used to establish that the deficiency spaces of a symmetric operator lie in the domain of its closure thus implying its essential self-adjointness. This idea in different situations (with different scales of regularity) was used to prove essential self-adjointness of globally hypoelliptic operators in $L^2(\mathbf{R}^n)$ ([7,44]), uniformly hypoelliptic operators in $L^2(\mathbf{R}^n)$ [28] and in Besikovich Hilbert space of almost periodic functions $B^2(\mathbf{R}^n)$ [29] , random elliptic operators in a Hilbert space of homogeneous random fields [46,49], uniformly elliptic operators on Lie groups [70,77]. It was noticed in [32], that a formal transfer to matrix operators permits to deduce from here the coincidence of minimal and maximal (or strong and weak) extensions for the considered operators. In [96] (see also [106]) some estimates of Green functions obtained in [95] were used to prove the coincidence of minimal and maximal extensions in L^1 for uniformly elliptic operators with C^∞-bounded coefficients on manifolds of bounded geometry (the main difficulty here is that the exact regularity theorems and duality arguments do not work in L^1).

   The paper [125] gives a simple proof of an important result by I. Oleinik which directly relates essential self-adjointness of a Schrödinger operator on a Riemannian manifold with the classical completeness of an associated Hamiltonian system. In [131] this result was extended to magnetic Schrödinger operators on manifolds.

   The paper [132] contains a generalization of the Povzner-Wienholtz theorem on essential self-adjointness of semi-bounded below Schrödinger operators in $\mathbf{R}^n$ to the case of magnetic Schrödinger operators on an arbitrary complete Riemannian manifold. The proof uses a non-trivial Karcher construction of cut-off functions with a controlled gradient. The result holds under almost optimal local regularity requirement on the electric potential.

The paper [137] develops a new technique to prove essential self-adjointness of Schrödinger type operators in sections of hermitian vector bundles on manifolds which do not a priori have a Riemannian metric but have a smooth positive measure. A natural Riemannian metric comes from the principal symbol of the operator under study. Singular potentials are allowed. A refined Kato inequality improving the result of Hess, Schrader and Uhlenbrock is established and effectively applied.

7. **Operators with almost periodic coefficients** [19-23,26,27,29,30, 32-34, 36, 42, 43,45,47,48,63,110].
In [19,20,33] different algebras of almost periodic pseudo-differential operators and spaces of almost periodic functions were introduced. Such operators act also in usual Sobolev spaces and the regularity theorems for elliptic and hypoelliptic operators hold in both usual and almost periodic spaces with usual applications. A problem of correspondence of spectra of almost periodic operators in different spaces arises and it was solved in [32] for the usual space $L^2(\mathbf{R}^n)$ and Besikovich Hilbert space $B^2(\mathbf{R}^n)$ where the spectra coincide (this was proved by some approximation of almost periodic functions by functions with a compact support and vice versa).

It was noticed in [23,33] that the II_∞-factor introduced by Coburn, Moyer and Singer in 1973 can be used to define and examine a distribution function of the spectrum for a self-adjoint almost periodic operator. This function has the set of points of increasing coinciding with the spectrum, and answering a question of F.A.Berezin it was proved in [26,42] that it coincides with the integrated density of states for general elliptic self-adjoint almost periodic operators. The integrated density of states is defined by a limit procedure from usual distribution functions of eigenvalues in bounded domains. The existence of the limit and of many similar ones (defining other spectral invariants like Fermi energy etc.) was proved in [42,47]. Also some asymptotics of the integrated density of states were obtained in [33,34,47]. One of them which is valid for the Schrödinger operator with an almost periodic potential has an estimate of remainder which is much better than the best possible Hörmander estimate concerning the case of elliptic operators on closed manifolds.

The structure of inverse operators for almost periodic pseudo-differential operators of different classes, complex powers and zeta-function for these operators were investigated in [22,33,43,47]. A generalization of the Favard theory to multidimensional case given by E.Muhamadiev plays an important role here. A generalization of the Favard theory which was given in [43,48] localizes the Favard almost periodicity condition to the points of the Bohr compact and provides conditions of continuity of a solution of a partial differential or even pseudo-differential equation in a single point of the Bohr compact.

An elementary proof of the index formula was suggested in [26,47]. It is based on the Muhamadiev approximation of the almost periodic coefficients by periodic ones on big cubes the size of which is an integer multiple of the period.

Other results on almost periodic operators are: stabilization of solutions for some natural class of almost periodic parabolic equations in the Besikovich norm [21] and essentiality of the spectrum of almost periodic pseudo-differential operators in $L^2(\mathbf{R}^n)$ [33] and $B^2(\mathbf{R}^n)$ [63].

A new approach to the spectral theory of the Almost Mathieu operator is suggested in [110]. It relates the spectrum with the spectrum of the Discrete Magnetic Laplacian which is a 2-dimensional discrete operator and an element in a II_1-factor. The latter fact simplifies working with the integrated density of states which is then expressed in terms of the trace in the factor. A sufficient condition is given for the spectrum to be a Cantor set. It is formulated in purely algebraic terms.

8. **Random elliptic operators** [35,38-41,46,49,57,61,66,78,82].
In [38] some general results of spectral theory were formulated which generalize to random elliptic operators of arbitrary order the corresponding results which were known for operators with almost periodic coefficients and for Schrödinger operators with random homogeneous potentials.

In [39-41] the index theory of random elliptic operators and their families was constructed. The index of a random elliptic operator here is a random real variable (in ergodic case it is just a real number) which is obtained by measuring of kernel and cokernel by means of taking of a random trace of the corresponding projections (where the random trace is defined as the mean value of the L.Schwartz kernel over the diagonal). The index formula which was given here generalizes the one for almost periodic operators. Moreover the case of families of random elliptic operators with parameters on a compact manifold was investigated and the index as a cohomology class of the parameter manifold was introduced and calculated giving a new result for the almost periodic case too. An application: the proof of the existence of an infinite-dimensional kernel or cokernel for some boundary value problems in a halfspace.

In [46,49] a natural action of random elliptic operators on spaces of homogeneous random fields was introduced. The usual pseudo-differential and Sobolev spaces technique was developed and used to prove essential self-adjointness results and a result on stabilization of solutions of parabolic equations.

In [57,59,66] the problem of coincidence of spectra for random elliptic operators in the usual L^2-space and in the Hilbert space of homogeneous vector fields was investigated. A functional calculus of L.Schwartz functions of uniform pseudo-differential operators was used to prove the coincidence in case when the dynamic system defining the homogeneous random coefficients field is aperiodic i.e. when its group of periods is trivial. In case of a non-trivial group of periods the spectrum in the space of homogeneous random field coincides with the spectrum of the same operator in L^2-space on the quotient space of $\mathbf{R}^n$ with respect to the group of periods.

In [61,78] a variational principle for the integrated density of states of self-adjoint random operators of arbitrary order was proved. The proof uses the index theory of random elliptic operators. The variational principle was used to prove an asymptotic formula for the integrated density of states of general hypoelliptic random pseudo-differential operators with an estimate of remainder by means of the method of approximate spectral projection.

9. **Transversally elliptic operators** [50,58,68,71].
In [50,58] a distribution function of the spectrum of a self-adjoint transversally elliptic operator on a compact manifold with an action of a compact Lie group was constructed. This function maps the spectral parameter line to the space of distributions on the Lie group which are constant on the classes of conjugate elements i.e. invariant with respect to the inner automorphisms. The value of this function at a point of the spectral line is the properly defined distributional character of the representation of the symmetry group in the image of the spectral projection corresponding to the chosen value of the spectral parameter. This function is increasing with respect to a natural partial order in the distributions defined by the cone generated by the characters of representations; the set of points of increasing coincides with the spectrum. The singularities of the values of the distribution function lie in the set of the elements of the group which have a non-empty set of fixed points and the order of the singularities may be estimated in terms of the Sobolev scale on the Lie group.

   In [68,71] a zeta-function of the transversally elliptic operator was defined also with values in the distributions on the Lie group. It was proved by use of the resolution of singularities of the action of the Lie group that this zeta-function has a meromorphic continuation to the whole complex plane with the poles on a finite number of rational arithmetic progressions. The residues of the poles are distributions on the Lie group with a singular support lying in the set of elements with a non-empty set of fixed points.

10. **Pseudo-differential operators on Lie groups** [70,74,77,84].
Algebras of uniform pseudo-differential operators were constructed and the corresponding Sobolev spaces technique was developed on unimodular Lie groups. The most important difficulty arising here is that the absence of the global symbols leads to the necessity of simultaneous requirements on local symbols and the global L.Schwartz kernels. The requirements on the kernels is their decay off the diagonal. The choice of the decay requirements may be done in many different ways thus leading to different classes of uniform pseudo-differential operators. The functional calculus of pseudo-differential operators in these classes was developed including the theory of complex powers of such

operators provided the Agmon condition on the symbol is satisfied. Some estimates of the Green functions were given. They depend on the behaviour of the kernel of the operator which is investigated and also on the asymptotic behaviour of the volume of the ball when its radius tends to infinity (this asymptotic may be either power or exponential). It is proved in particular that for the elliptic differential operators the Green function always decays exponentially.

11. **Pseudo-difference operators and their Green function** [69,72]. Pseudo-difference operators are operators on a discrete metric space with some estimates of the matrix elements, especially estimates which assume a decay of them off the diagonal. Algebras of pseudo-difference operators are constructed and estimates of the Green function for the invertible operators are given. In particular the coincidence of spectra of a pseudo-difference operator in different spaces L^p is proved provided the number of points in a ball of the radius R grows subexponentially if R tends to infinity.

12. **Complete asymptotic expansion of spectral invariants** [54,62,65,73,76].
    In [53] the complete asymptotic expansion of the spectral shift function for elliptic systems having constant coefficients near infinity is obtained provided the non-trapping condition is satisfied.

    In [62,65] the complete asymptotic expansion of the spectral function near the diagonal is obtained for the elliptic operators of the second order with constant coefficients near infinity provided the non-trapping condition for the rays is satisfied. (Later B.R.Vainberg improved this result extending this asymptotics to all points which may be far from the diagonal).

    The main idea in [54,62,65] is to use the Fourier transform passing to a corresponding hyperbolic problem and then use the local decay of energy to perform the inverse Fourier transform. This idea does not work for the Hill operators which were considered in [73,76]. There specific one-dimensional methods were used to prove the complete asymptotic expansion of the spectral function for the Hill operators (one-dimensional Schrödinger operators with periodic potentials). There is no local decay of energy in this situation. Therefore a special feature of the asymptotics arises in direct contrast with the case of operators with constant coefficients near infinity: the asymptotics cannot be differentiated with respect to the spectral parameter even once. More exactly singularities of the first derivative arise near the ends of the forbidden zones (lacunas) of the spectrum.

13. **Non-standard analysis and singular perturbations of ordinary differential equations** [67].
The existence of ducks in case of a degenerate fold point on the slow curve of a two-dimensional fast-slow vector field is proved. Here ducks are trajectories of the fast-slow vector field that go some way along the stable part of the slow curve and then pass to the unstable part and go along this part during some non-infinitesimal time. The ducks occur for the fast-slow fields which depend on an additional parameter except the usual infinitesimal one. Also another proof of the existence of the asymptotic expansion of ducks and the duck parameter was given by use of the exponential microscope. The investigation of ducks is done by methods of the non-standard analysis first used and developed in these problems by French mathematicians G.Reeb, E.Benoit, J.-L.Callot, F.Diener, M.Diener and others.

14. **Elliptic operators on manifolds of bounded geometry** [85,86, 95,100,103,106,127,128,130].
In [85,86,103] elliptic self-adjoint operators on normal covering manifolds of compact manifolds were studied. For such operators the distribution function of the spectrum can be introduced either analytically or by use of a von Neumann dimension. A variational principle for this function was proved and applied to obtain some asymptotics of this function with an estimate of remainder which is sometimes much better than usual estimates in the case of the discrete spectrum. (An example is the operators of Schrödinger type on the hyperbolic space with potentials which are periodic with respect to a discrete group of isometries of the space with a quotient of a finite volume.)

    In [95,106] a Schnol theorem was generalized to the case of manifolds of bounded geometry. The original Schnol theorem claims that if a Schrödinger operator with a semi-bounded from below potential has a non-trivial eigenfunction of a subexponential growth then the corresponding eigenvalue belongs to the spectrum of the operator in $L^2(\mathbf{R}^n)$. It was noticed by Kobayashi, Ono and Sunada that the same is true for periodic Schrödinger operator on a covering of a compact manifold provided it has a subexponential growth of the volume function. Here the subexponential growth condition is essential as is seen if we consider the Laplacian on the hyperbolic space. In [95] the same statement was obtained for general elliptic operators with bounded coefficients on subexponential manifolds of bounded geometry were it was deduced from the exponential decay of the Green function. In [106] the result was extended to manifolds without subexponential growth; in this case the subexponential growth condition for the eigenfunction should be replaced by the condition that the eigenfunction multiplied by $\exp(-\epsilon \mathrm{dist}(x, x_0))$ belongs to L^2. The same statement is true if L^2 is replaced by L^p.

For a Schrödinger operator with a semi-bounded below potential on a manifold of bounded geometry a necessary and sufficient condition of the discreteness of the spectrum in terms of the potential is given in [127,128,130]. This condition is formulated in terms of the Wiener capacity in geodesic coordinates. This result generalizes the famous result of A.M.Molchanov (1953) who considered the Schrödinger operators on the flat (Euclidean) space.

15. **Non-linear equations** [56,75,83,91,143,146].
For non-linear integrable evolution equation of the Korteweg – de Vries hierarchy asymptotic solutions which grow at infinity with respect to the space variable were constructed in [56]. (They are solutions of the Cauchy problem modulo the L.Schwartz functions). The question arises whether these solutions are really asymptotics of the exact solutions. The answer was later given by I.Bondareva for the equations of the Korteweg – de Vries (KdV) type but with arbitrary first order terms. In [75] the uniqueness of the solution of the Cauchy problem in these classes of growing functions was proved. In [83,91] the existence classes were enlarged to include growing functions with some estimates but with no asymptotics at infinity. (The uniqueness of solution in these classes follows from the results of [75]).

In [143,146] the Miura transform on the line was studied and used to provide existence and uniqueness results in new classes of functions and distributions for the Cauchy problem for KdV and mKdV (modified KdV) equations. The Miura transform is a non-linear quadratic map which maps solutions of mKdV to solutions of mKdV. This map is not one-to-one, but a new algebraic machinery allows to use this transform, for example, to prove existence and uniqueness for mKdV in classes of functions which are unbounded at infinity, i.e. extend the results of [56,75,83,91] from KdV to mKdV.

16. **Lefschetz-type formulas** [52,93,94,97,98].

    A Lefschetz fixed point formula for elliptic complexes of Boutet – de-Monvel operators on a compact manifold with boundary is proved in [52,98]. It generalizes to the case of manifolds with boundary the well known result of Atiyah and Bott but the proof is much more complicated technically. An interesting feature of the obtained formula is that a classification of the boundary fixed points is involved; namely, they are divided into two parts: attracting and repelling ones. For example, if we consider the classical De Rham complex then only attracting boundary fixed points give non-zero contributions in the Lefschetz fixed point formula in absolute cohomologies, but the situation in relative cohomologies is opposite: only repelling fixed points matter.

    In [93,94,97] a Lefschetz fixed point formula in L^2-cohomologies on manifolds with cylindric ends is obtained provided the ends have spherical bases. An interesting feature of the result is that the final answer contains not only contributions of the fixed points but also some global invariants like degrees of some maps defined by the map under study and its asymptotics at infinity.

17. **Von Neumann algebras and topology of non-simply connected manifolds** [79-81,86,101,102,104, 112, 113, 117,121].

    In [79-81] the real von Neumann Betti numbers introduced by Atiyah were used in Morse inequalities on non-simply connected manifolds. Examples were given that in some cases they really improve the classical Morse inequalities. A Witten-type proof of these Morse inequalities and of L^2 version of the Novikov inequalities for vector fields was given in [113] by use of a very general theorem on semi-classical asymptotics of spectra of periodic operators on covering manifolds. Similar Novikov-type inequalities (for closed 1-forms which are interpreted as "multi-valued" Morse functions) were proved in [117].

Some analytic invariants of vector fields on manifolds were introduced in [112] based on Novikov inequalities for vector fields and use of Witten type deformation of the De Rham complex associated with the given vector field. The idea is to take background values of the dimension of the null-space for corresponding Laplacian and consider this dimension as a function on the set of all Riemannian metrics. In this way we get an analytic stratification of the (infinite-dimensional) space of all metrics. All invariants of this stratification will be invariants of the vector field. If we vary the vector field itself then we obtain a stratification on the space of all vector fields which gives us invariants of the underlying manifold.

In [81] new invariants of non-simply connected manifolds are constructed. (They were later named Novikov-Shubin invariants.) The idea is that the asymptotics of the von Neumann trace of the heat operator on differential forms near infinity does not depend on the Riemannian metric and is therefore an invariant of the manifold (the proof was published in [86]). It was later proved in a joint paper by M.Gromov and M.A.Shubin [102] that these invariants are in fact homotopy invariants. Moreover in [102,104] they were expressed in terms of some new invariants which are defined not only for closed manifolds but also for manifolds with boundary and even for general finite CW-complexes. It is done by means of a general construction of some chain homotopy invariants of Hilbert complexes with an additional non Neumann structure. These invariants are constructed by a formal application of the variational principle for the spectrum distribution function but on the other hand they have a geometric interpretation as some asymptotic dimensions of "near–cohomologies" obtained by taking cochains with small coboundaries modulo cochains close to cocycles.

M.Farber interpreted these invariants together with the von Neumann Betti numbers in terms of a new cohomology theory which he called "extended cohomology". In [121] de Rham theorem for extended cohomology is proved.

18. **Idempotent analysis** [107].

Homomorphisms of idempotent semirings with division are described in terms of their multiplicative kernels i.e. kernels of induced homomorphisms of multiplicative groups. Such a kernel may be any subgroup which is convex in a natural sense.

An analogue of the L.Schwartz Kernel Theorem is proved in spaces of all bounded functions on arbitrary sets with values in a boundedly complete idempotent semiring.

19. **The Riemann–Roch theorem for general elliptic operators** [105,108,109,111].
A generalization of the classical Riemann–Roch theorem is proved in [108]. It relates two dimensions: the dimension of the space of the solutions of an elliptic equation (in sections of vector bundles) with possible poles and zeros at prescribed finite set with given multiplicities (a "point divisor"), and the corresponding dimension for the inverse divisor and the transposed (or adjoint) operator. An important corollary is an inequality which gives a lower bound for the dimension implying e.g. the existence of non-trivial solutions with a prescribed finite set of zeros if a pole of a sufficiently high order is allowed. If the transposed operator has the unique continuation property (i.e. there is no non-trivial solutions with zeros of infinite order) then the dimension of the space of solutions corresponding to a divisor can be explicitly calculated provided the number of poles (multiplicities counted) is much greater than the number of zeros.

In [109] this result was extended to the situation when singularities can be distributed on a compact nowhere dense set, and vanishing conditions are replaced by orthogonality conditions to some distributions taken from a finite-dimensional space and supported on another compact nowhere dense set. A corollary of this result gives an approximate solution of the Cauchy problem for elliptic equations with initial conditions on a compact nowhere dense set. Here approximation means that the Cauchy initial conditions are satisfied not precisely but up to a function (or section) which is orthogonal to any given finite-dimensional space of distributions.

In [111] an L^2-version of the Riemann-Roch theorem is proved for elliptic operators on normal covering manifolds. It implies existence results for L^2-solutions with singularities.

20. **L^2-holomorphic functions** [120, 122].
Application of the von Neumann algebra technique to the Dolbeault complex on coverings of pseudoconvex compact complex manifolds leads to an extension of the classical Oka-Grauert theory to the case of coverings. In particular, the existence of an infinite-dimensional space of L^2 holomorphic functions can be proved on any regular covering of a compact strongly pseudo-convex manifold.

    An example is given which shows that bounded-geometry-type conditions are not sufficient to insure the existence of non-trivial L^2 holomorphic functions. In this example a complex manifold M with a strongly pseudoconvex boundary is constructed such that there exists a free holomorphic action of a 3-dimensional solvable Lie group G on M with $M/G = [-1, 1]$, but there are no non-trivial L^2 holomorphic functions on M. The reason is that the group G is not unimodular which makes it impossible to apply von Neumann algebras type II technique. Therefore this topic gives an example of a situation where application of the von Neumann algebras in analysis is truly essential.

21. **Geometry of symplectic connections** [123, 128].
Geometry of symplectic connections on manifolds is studied in [123]. In particular, a generalization of the classical Levi-Civita construction of the Riemannian connection is given. Curvature of symplectic connections is studied. The Thomas-Veblen theory of normal tensors and extensions is developed for Fedosov manifolds (symplectic manifolds with a fixed symplectic connection). Necessary and sufficient algebraic conditions are obtained for tensors which can be curvature tensors of symplectic connections at a point and also the first covariant derivatives of such tensors.

    A natural sequence of connections associated with a Riemannian and almost symplectic structures is studied in [128]. In particular, a new characterization of Kähler manifolds is given in terms of this sequence.

22. **Elliptic boundary problems with relaxed conditions** [124].
The Fredholm property of the operator associated with an elliptic boundary problem on a compact manifold with boundary was used

to prove explicit solvability results for the "relaxed" problem where the equation or the boundary conditions are "relaxed" (not required to hold) on a non-empty open set.

The Breuer version of the Fredholm property (in an appropriate von Neumann algebra) is established in uniform Sobolev L^2 spaces for the case when the problem is considered on a non-compact manifold M (with boundary) with a free action of a discrete group Γ such that M/Γ is compact. Existence of infinite-dimensional space of solutions of the problem is deduced if zero conditions are imposed outside of an open Γ-invariant subset of the boundary.

23. **Spectra of magnetic Schrödinger operators** [133, 136, 140, 141].

    It is well known that the condition $V(x) \to +\infty$ as $x \to \infty$ guarantees the discreteness of spectrum for the Schrödinger operator $-\Delta + V(x)$ in $L^2(\mathbf{R}^n)$ (K.Friedrichs, 1934). If V is semi-bounded below, then even a necessary and sufficient condition for the spectrum to be discrete can be formulated in terms of the Wiener capacity (A.M.Molchanov, 1953). For the magnetic Schrödinger operators some Friedrichs type results were known when there is no electric field (J.Avron, I.Herbst and B.Simon, 1978; A.Dufresnoy, 1983; A.Iwatsuka, 1990). The paper [133] is the first to provide Friedrichs type sufficient conditions for the discreteness of spectrum such that they take into account a combined action of electric and magnetic fields. These conditions have the form $V_{eff}(x) \to +\infty$ as $x \to \infty$ where the effective potential V_{eff} is an explicit combination of electric and magnetic potentials. The paper also contains stronger results which use the capacity and give sufficient conditions of the discreteness of spectrum which become also necessary if the magnetic field vanishes.

    A family of necessary and sufficient conditions of the discreteness of spectrum and strict positivity for magnetic Schrödinger operators with positive scalar potentials was obtained in [140]. These conditions depend on a functional parameter. It follows from the main result that these conditions are equivalent.

    In [136] an L^2 version of the semiclassical approximation of magnetic Schrödinger operators with invariant Morse type scalar potentials on covering spaces of compact manifolds is studied. In particular, the existence of an arbitrarily large number of gaps in the spectrum of these operators is established in the semiclassical limit as the coupling constant goes to zero.

The paper [141] provides a new method of finding semiclassical asymptotics of spectra in this and similar situations. It gives a new proof of the above formulated result and strenghtens it by providing a construction of Murray - von Neumann equivalence of the spectral projections with the same spectral parameter for the original magnetic Hamiltonian and for the model operator which is a direct sum of quantum harmonic oscillators corresponding to the bottoms of the scalar potential wells. This equivalence entails vanishing of higher traces in cyclic cohomology for the spectral projections, and in particular implies vanishing of the quantum Hall conductivity for low energies.

24. **Negligible sets in spectral theory of Schrödinger operators** [142, 144]. A well known result of A. Molchanov (1953) gives a necessary and sufficient condition for the discreteness of spectrum of the Schrödinger operators with positive potentials. This condition reveals that the behavior of the potential can be ignored on some subsets ("negligible" sets) in cubes of a fixed size. Molchanov described negligible sets as sets which have small capacity, compared with the capacity of the cubes. It is observed in [142] that the notion of negligibility can be considerably expanded so as to include sets whose capacity constitutes e.g. 99% of the capacity of the cubes. The corresponding condition on the potential proved to be equivalent to the condition when the sets are considered negligible when they constitute 1% of the capacity of the cubes. This equivalence is a non-trivial property of capacity; it is deduced from the fact that both conditions are equivalent to the discreteness of spectrum. A similar situation appears with necessary and sufficient conditions of strict positivity. The results of [142] in particular answer a question asked by I.M. Gelfand in 1953.

    Similar ideas were used in [144] to produce two-sided estimates of the bottom of the spectrum of the Dirichlet Laplacian in a domain of the Euclidean space, in terms of the capacitary interior radius of this domain. Here again the same classes of negligible sets are used. The main result can be considered as a partial answer to the question "Can one see the fundamental frequency of a drum?"

25. **Geometric theory of lattice vibrations and specific heat** [145]. The asymptotic behavior of the specific heat of a solid at small temperatures is discussed from the point of view of spectral theory of crystal lattices. The subject is classical in solid state physics. (It goes back to Einstein (1907) and Debye (1912).) Using a special quantization of crystal lattices and calculating the asymptotic of the integrated density of states at the bottom of the spectrum, we obtain a rigorous derivation of the classical Debye T^3 law. The idea and method are taken from discrete geometric analysis which has been recently developed for the spectral geometry of crystal lattices.